

Emergence of power-law correlation in 1-dimensional self-gravitating system

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Abstract

A new phase of temporal evolution of the one-dimensional self-gravitating system is numerically discovered. Fractal structure is dynamically created from non-fractal initial conditions. Implication to astrophysics and mathematical physics is discussed.

1 Introduction

1-dimensional self-gravitating system is useful for describing the dynamics of self-gravitating many body systems such as globular clusters, elliptical galaxies, and so on [HF67, RM88, YGS92, TKG97]. The model represents the motion of parallel flat mass sheets interacting each other through Newtonian gravity.

Yamashiro et al. studied this model numerically to investigate relaxation process of elliptical galaxies [YGS92]. According to the virial theorem the virial ratio $V_r \equiv 2E_{kin}/E_{pot}$ for this model is expected to relax to 1. They chose initial conditions with $10^{-3} \leq V_r \leq 1$. Typical feature of the system they described is shown in Fig. 1, where a snapshot of one particle distribution (x_i, u_i) is shown. We can see a big cluster is formed.

While their main concern was the process of relaxation, they also noted that initial conditions with smaller virial ratio (typically $V_r < 10^{-3}$) did not guarantee the relaxation within their CPU time. They also observed

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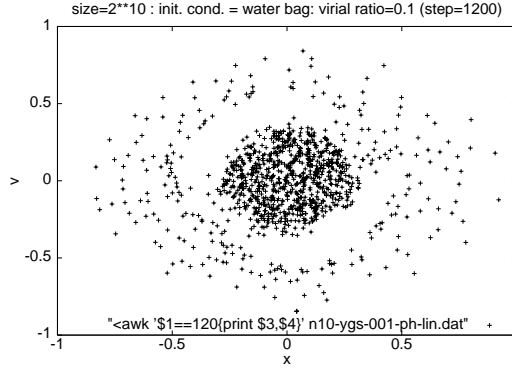


Figure 1: One particle distribution of the model 1 for $N = 2^{10}$, $V_r = 0.1$. One big cluster is formed and fractal structure is not present. $E=0.25$, Time=9.375.

formation of lumped structure as observed in [HF67], which are quite different from the one shown in Fig. 1.

We extend their study to the region $V_r \rightarrow 0$ and found that there is a new phase of evolution where a fractal spatial structure is spontaneously created. We would like to stress that the fractal structure is created even when the initial condition is not fractal (typically uniformly random). We also note that not all the initial conditions lead to fractal structure.

In this letter we report the discovery of this novel phase. In the next section we describe the model. We present the numerical results in section 3, and the final section is for summary and discussions.

2 Model

The one-dimensional self-gravitating system (also known as ‘mass-sheet model’) consists of N equivalent flat sheets of constant mass density. The sheets are infinitely extended in the y and z direction and are aligned parallel to each other and move in the x direction.

If we suppose the sheets interact with Newtonian gravity, the Hamiltonian of the system reads

$$H = \sum_{i=1}^N \frac{p_i^2}{2m} + 2\pi G m^2 \sum_{i>j} |x_i - x_j|, \quad (1)$$

where $p_i = mu_i$. (u_i is the velocity of the sheet i .) We set $m = 1/N$ so that the total mass is unity, and $4\pi G \equiv 1$.

With this Hamiltonian, the equation of motion reads

$$\ddot{x}_i = \frac{1}{2N} (N_R - N_L) \quad (2)$$

where N_R and N_L are number of sheets which are on the right and left of the sheet i , respectively.

Eq. (2) indicates that the force acting on each sheet is constant until a collision occurs between two sheets. Hence we can exactly integrate the equation of motion (2) between every two collisions. This fact greatly enhances the usefulness of this model.

3 Numerical results : structure formation

Fig. 2 show the evolution of the system (1) with $N = 2^{15}$. Initial condition is chosen to be $V_r = 0$ as $x_i =$ uniformly random $\in [0, 1]$ and $u_i = 0$. That is, each sheet is placed randomly with zero velocity dispersion. The figures represent one-particle distribution in (x, u) space. In the course of time evolution we see that fractal structure is formed.

Fig. 3 is a two-body correlation function $\xi(r)$ at $t = 9.375$ in Fig. 2. $\xi(r)$ is defined as

$$dP = ndV(1 + \xi(r)) \quad (3)$$

where dP is probability to find a sheet in a volume dV at distance r apart from a sheet, and n is the average number density. We can see clear power-law behavior in $\xi(r)$ in Fig. 3.

This fractal structure is also formed if we modify the initial condition as $x_i =$ uniformly random $\in [0, 1]$ and $u_i = a \sin 2\pi x_i$, as shown in Fig. 4.

If we slightly increase the virial ratio from 0, the fractal structure is not created. Fig. 5 show the 1-dimensional phase space for an initial condition $x_i =$ uniformly random $\in [0, 1]$ and $u_i =$ uniformly random $\in [-0.01, 0.01]$. Here we see that fine structure observed in Fig. 2 is no longer observed. Flatness of two-point correlation function $\xi(r)$ in Fig. 6 supports the impression that structure here is not a fractal but just cramps with a constant density. This type of state is also observed in [HF67] and [YGS92].

The fractal structure seen in Fig. 2 is not a stationary state. As shown in Fig. 7, the fine structure once created is gradually dissolved into large clusters. The reason for the destruction of this structure is not clear now.

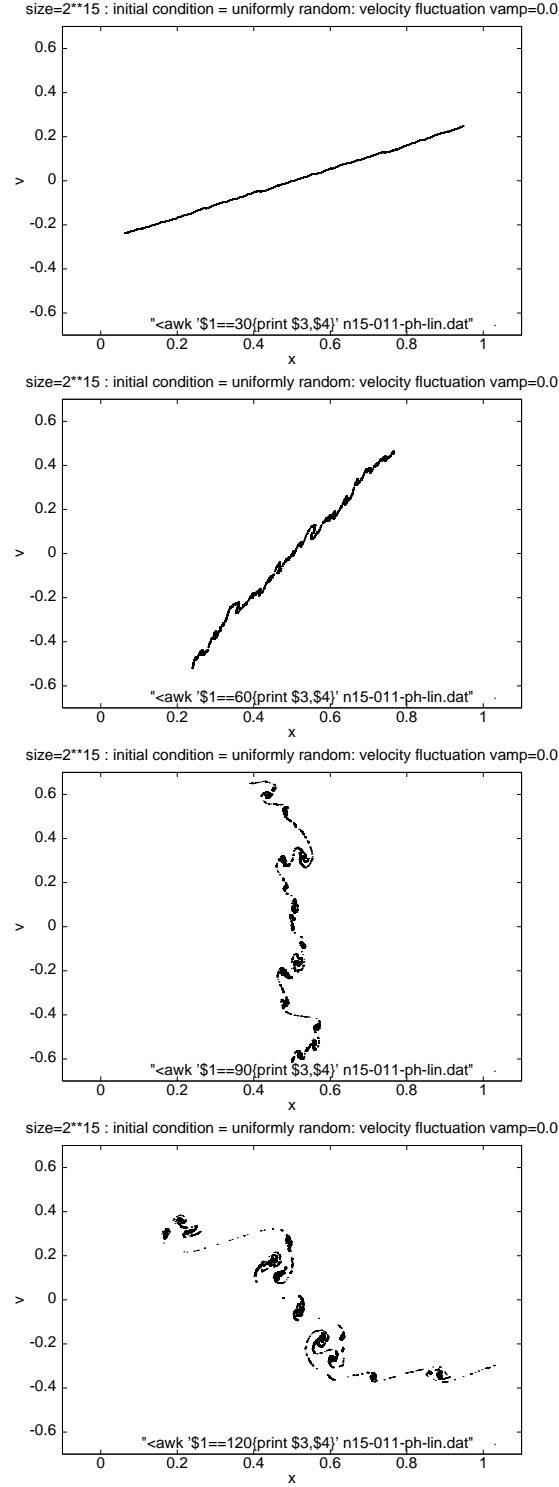


Figure 2: Formation of fractal structure. Initial condition : x_i : uniformly random in $[0,1]$, $u_i = 0$, $N = 2^{15}$. Time are 2.34375, 4.6875, 7.03125, and 9.375.

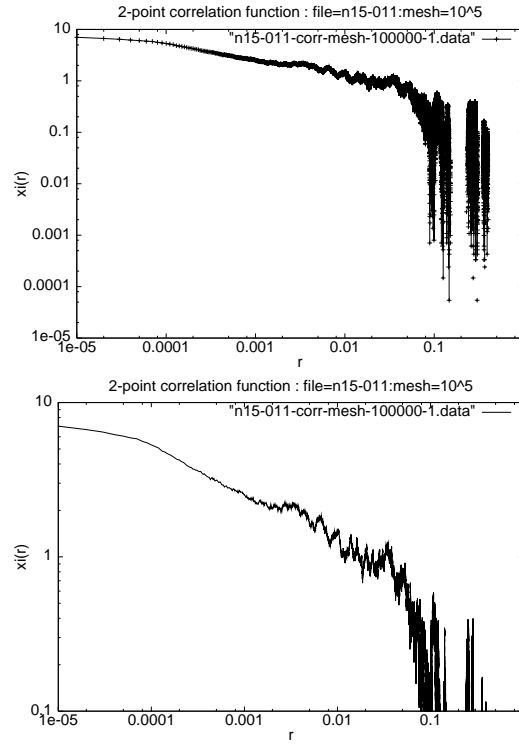


Figure 3: two-point correlation function $\xi(r)$ for $t = 9.375$ in Fig. 2. These two figures are identical except for the range of vertical axes.

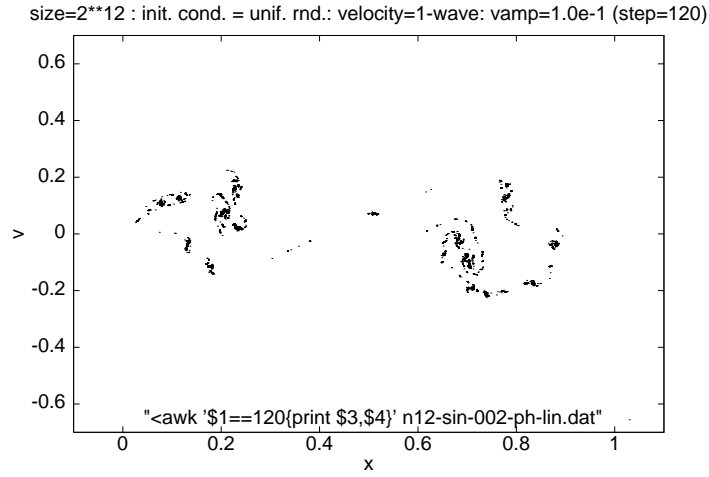


Figure 4: Fractal structure formed from initial condition x_i : uniformly random in $[0,1]$, $u_i = 0.1 \sin 2\pi x_i$, $N = 2^{12}$.

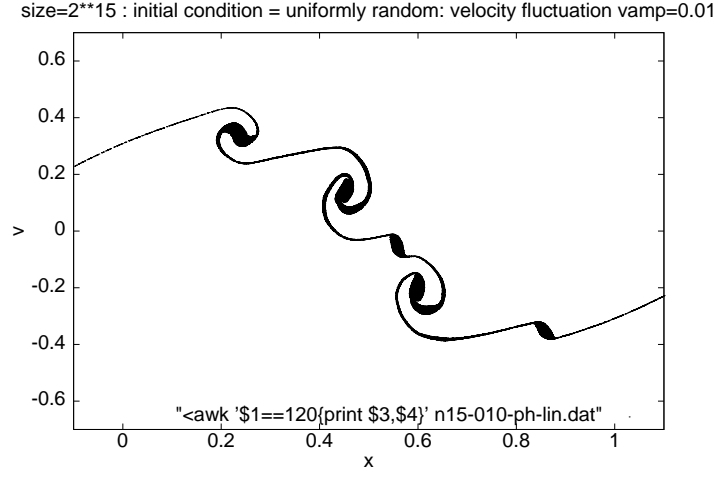


Figure 5: Example for $V_R \neq 0$. Fractal structure is not formed. Initial condition : x_i : uniformly random in $[0,1]$, $u_i = \text{random} \in [-0.01, 0.01]$, $N = 2^{15}$. Time = 9.375.

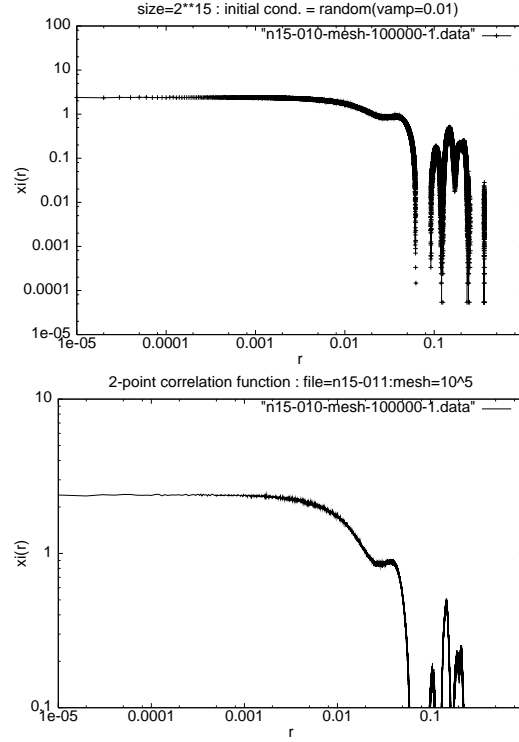


Figure 6: two-point correlation function $\xi(r)$ for $t = 9.375$ in Fig. 5. These two figures are identical except for the range of vertical axes.

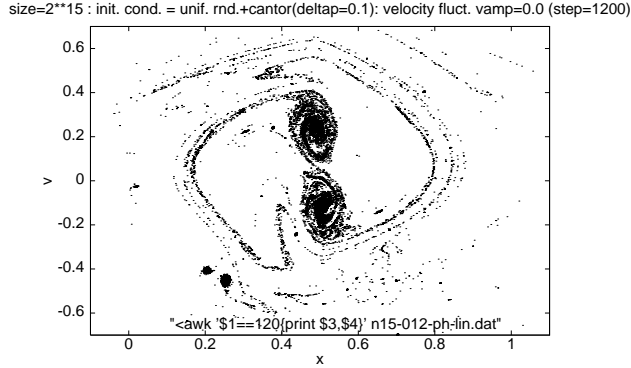


Figure 7: Late stage of Fig. 2. Time=93.75.

4 Summary and discussions

In this letter we have shown that there is a new phase in the evolution of one-dimensional self-gravitating system where the system creates fractal spatial structure from non-fractal initial conditions. This novel phase appears for initial conditions with small virial ratio. The structure is also formed for zero velocity dispersion (i.e., ‘cold’ condition). Other initial conditions with large virial ratio do not lead to this phase.

As far as we observed, this phase is not a stationary state. Whether or not this transiency is a real one or an artifact caused by, for example, our computational limitation (finiteness of number of particles and so on) is not clear now.

Several studies revealed that two-point correlation function of young stars in star-forming regions obey power-law [Lar95, NTHN98]. The fractal structure we have found may be accounted for an origin of the power-law. Also it is a subject of future study if the phase can be found in other systems, such as galaxies in the course of evolution.

Similar structure is observed in a model with expanding universe [TM00]. It will be an interesting subject to examine the effect of expanding universe on the structure formation.

We would like to stress the importance of the fact that, to create fractal structure, it is not necessary for initial conditions to be fractal. Fractal structure does not have characteristic spatial scale, nor does Newtonian potential. Hence it may not be so surprising that fractal structure can be found in systems interacting with Newtonian potential. However the relation between these two scale-free objects, Newtonian potential and fractal structure, are still unknown. If the initial condition is fractal, since the Newtonian interac-

tion does not have characteristic length scale, the self-similar property of the initial condition would be conserved. In fact, initial conditions with exact self similarity (e.g., Cantor set) evolve in time while keeping the self-similar spatial structure. On the other hand, we have found that non-fractal initial conditions also leads to fractal structure, hence in this case the structure is created by the dynamics itself.

The search for the mechanism why the fractal structure emerges will be quite interesting. The emergence is interesting because it is an asymptote to scale-free behavior. The emergence is also unique in that this is a structure formation in conservative system against the tendency to thermal relaxation [KK92, KK94].

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